LODZ UNIVERSITY of TECHNOLOGY
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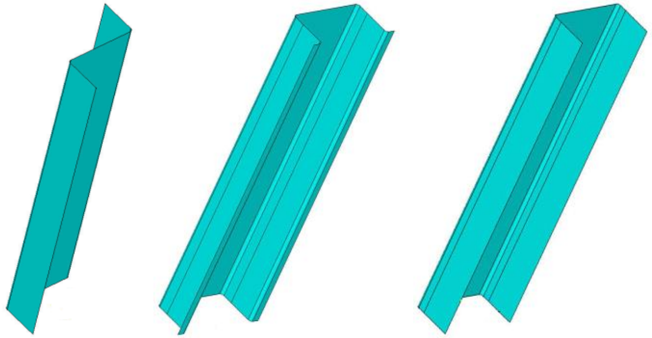
A STUDY on BUCKLING RESPONSE of FML MEMBERS of 'CLASSIC' VERSUS THIN-PLY DESIGN

VIBRATIONS AND BUCKLING
FACULTY OF ENGINEERING, UNIVERSITY OF PORTO
PORTO 7-9 MARCH 2016

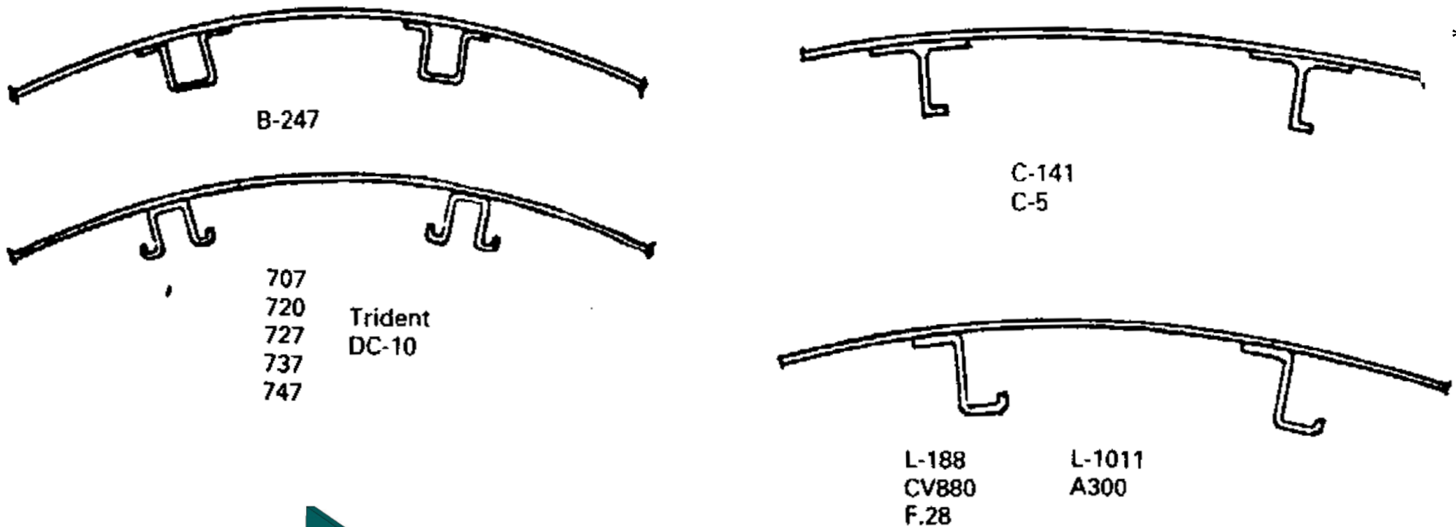
Subject of consideration



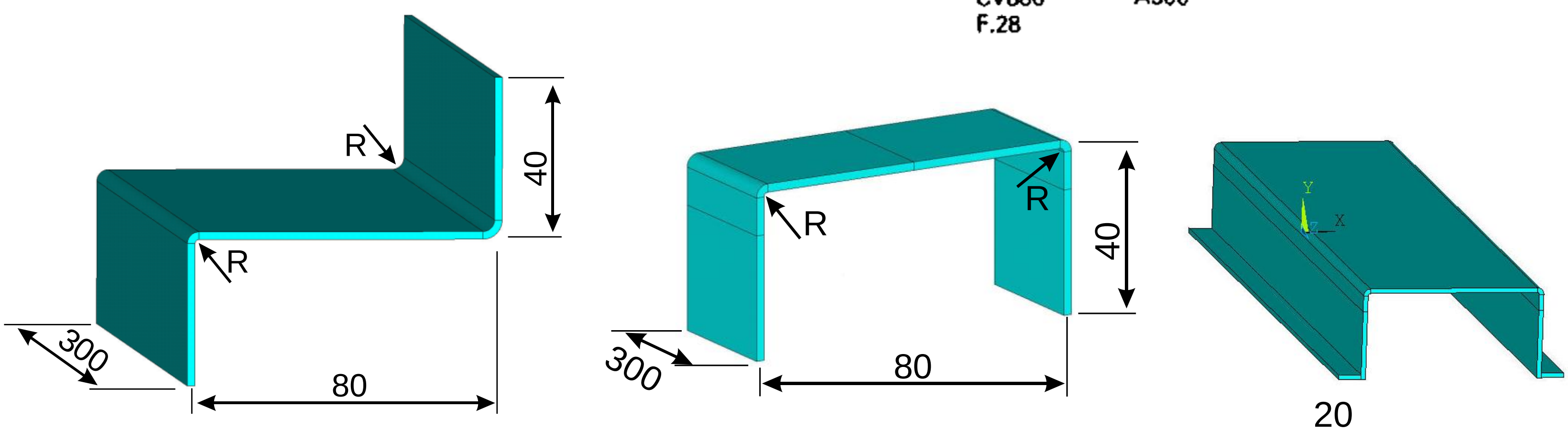
- 3 approaches of FML profiles buckling analysis.
- mechanical properties of components.
- eigen-buckling and non-linear post-buckling.
- experimental buckling.
- thin-ply design.
- conclusions.

This study is supported by the Ministry of Science and Higher Education in Poland – National Science Centre
Grant No **UMO-2012/07/B/ST8/04093**.

Thin-walled open cross-section stringers

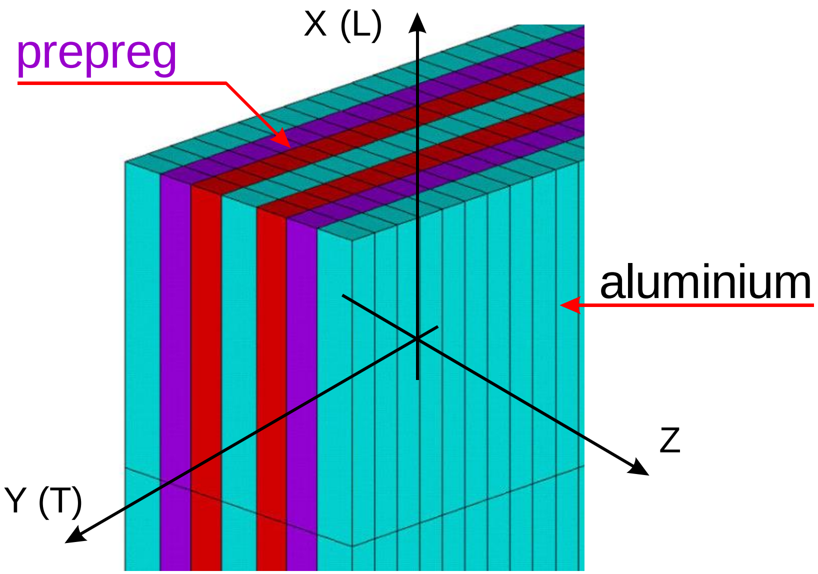


*)



*) M. Chun-Yung Niu – Airframe structural design. Technical Book Comp.. LA. Cal. 1988

Subject of consideration



Sequence	Lay-up
1[a]	Al/0/90/Al/90/0/Al
2	Al/90/0/Al/0/90/Al
3	Al/45/0/Al/0/45/Al
4	Al/0/45/Al/45/0/Al
5[b]	Al/0/0/Al/0/0/Al
6	Al/25/0/Al/0/25/Al
7	Al/0/25/Al/25/0/Al
8	Al/Al/Al/Al/Al/Al/Al
9	Al/*Iso*/*Iso*/Al/*Iso*/*Iso*/Al
10[c]	Al/45/-45/Al/-45/45/Al

t_{Al} = 0.3 mm. t_C = 0.25 mm

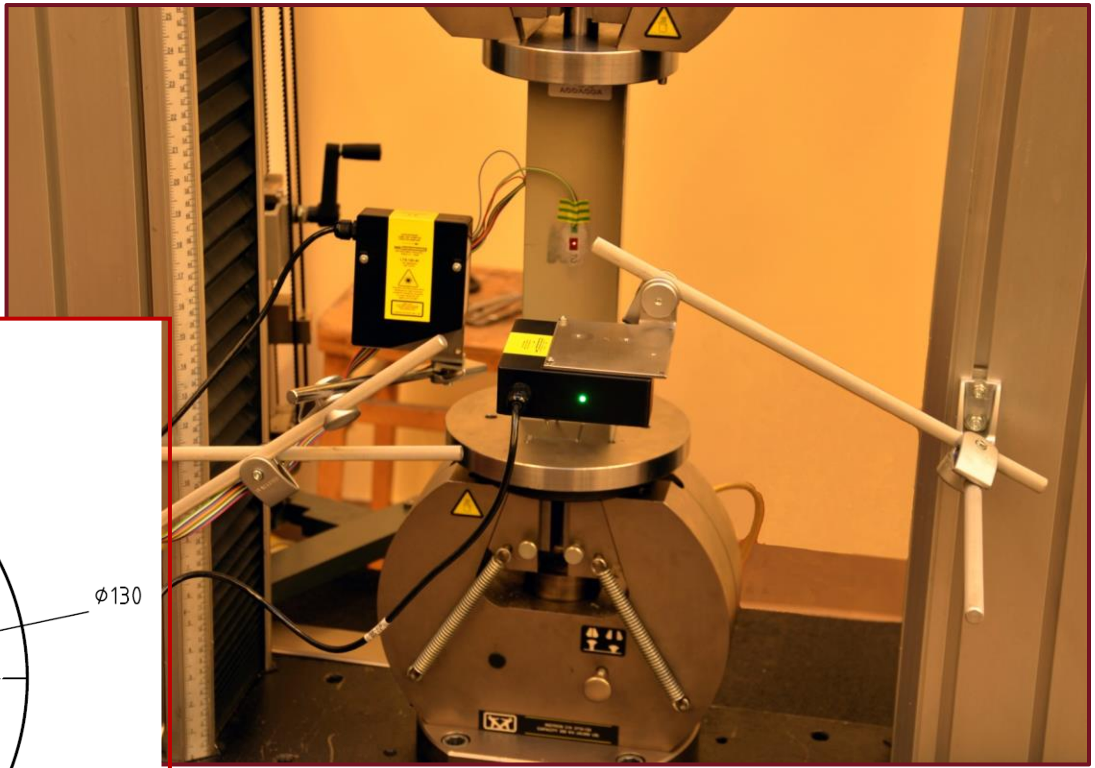
a – GLARE 3;
b – GLARE 2A;
c – GLARE 6A.

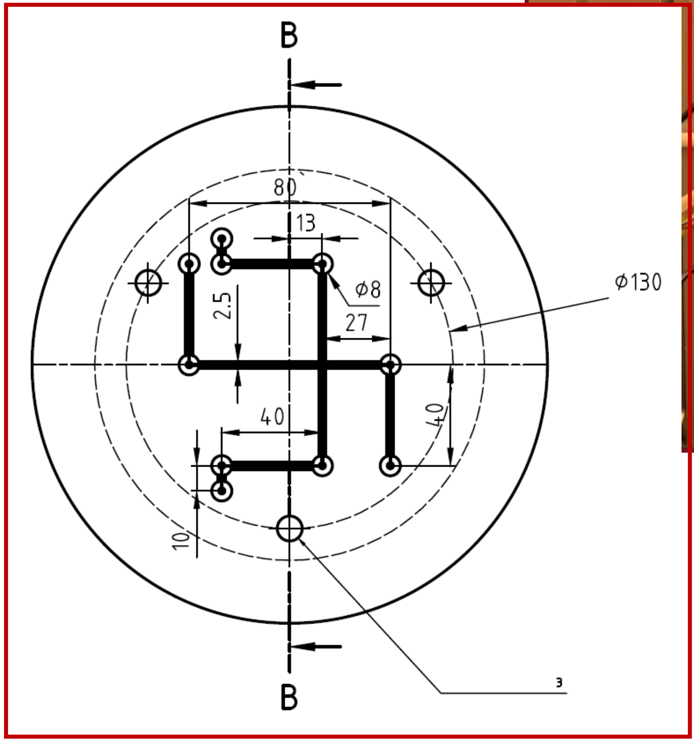
Material properties of FML components

	FML component property					
	Aluminium			TVR380		120EP-513/CF
[N/mm^2]	E	72×10^3	77×10^3	E_1	46.4×10^3	136.1×10^3
[ ]	ν	0.33	0.33	E_2	14.9×10^3	7.01×10^3
				G_{12}	5.2×10^3	4.661×10^3
				ν_{12}	0.269	0.274
[N/mm^2]	R_e *)	360	309	R_L	1534	2609
	R_m	448	408	R_T	74.5	nd
				S_L	1046	88.26
				C_L	115	869

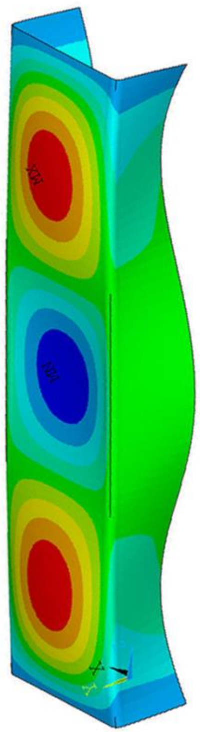
*) very small orthotropy of yield limit

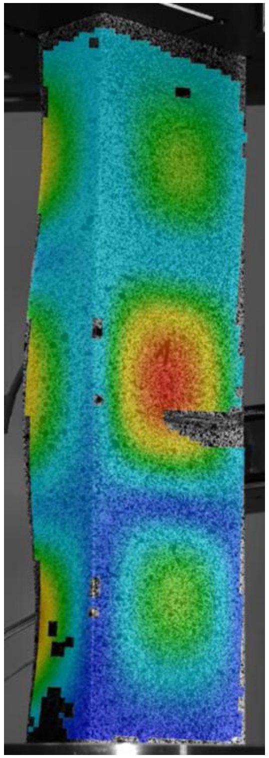
Test stand

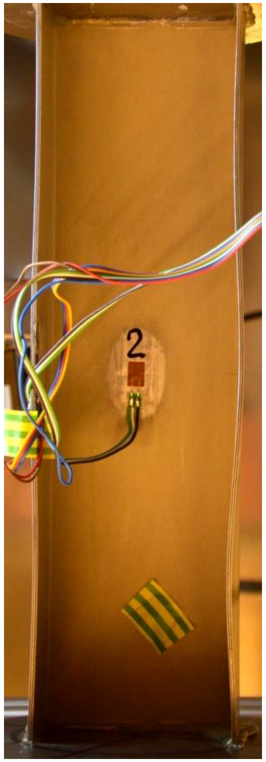




Buckling modes



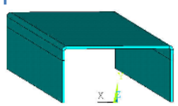




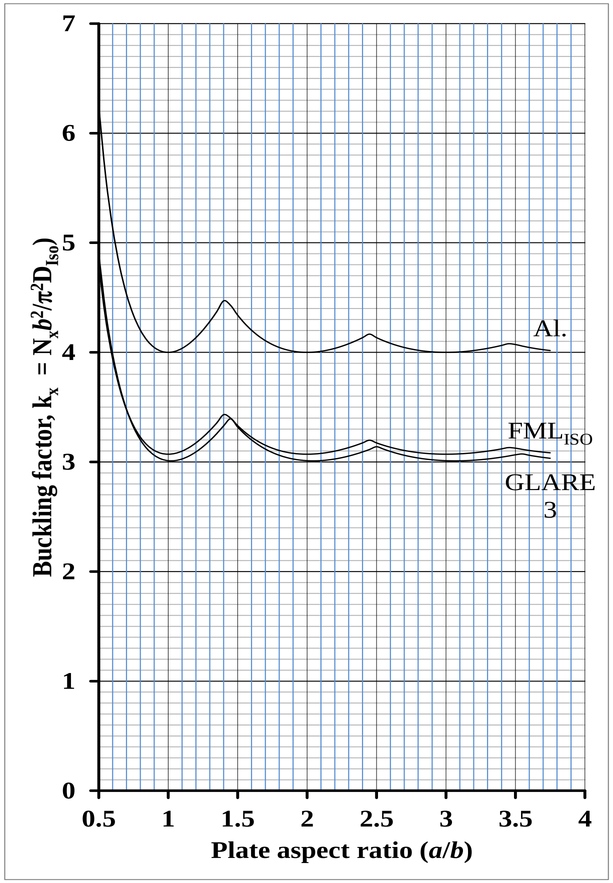
SHELL

EKSPERYMENT

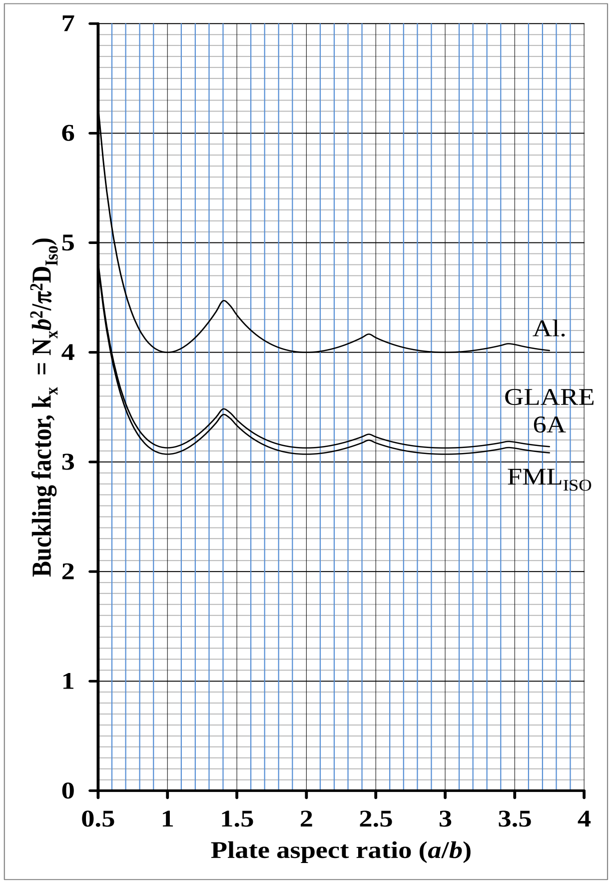
Buckling force as a function of GFR lay-up

Buckling force – channel section				
 Lay-up	exp [kN]	FEM [kN]	ANM Koiter [kN]	
AL/0/90/AL/90/0/AL	**31.434**	**30.189**	**28.568**	0.746
AL/90/0/AL/0/90/AL	**nd**	**29.871**	**28.408**	0.738
AL/45/0/AL/0/45/AL	**32.634**	**31.399**	**29.876**	
AL/0/45/AL/45/0/AL	**nd**	**30.588**	**29.015**	
AL/0/0/AL/0/0/AL	**29.836**	**30.310**	**28.630**	0.749
AL/25/0/AL/0/25/AL	**nd**	**30.745**	**29.334**	
AL/0/25/AL/25/0/AL	**29.856**	**30.977**	**28.859**	
Al/Al/Al/Al/Al/Al/Al	**nd**	**40.472**	**38.510**	1.
Al/*Iso*/*Iso*/Al/*Iso*/*Iso*/Al	**nd**	**30.805**	**29.311**	0.761
Al/45/-45/Al/-45/45/Al	**nd**	**31.752**	**30.208**	0.784

Buckling factor curves for rectangular plate



GLARE 3 $[Al/0/90/Al/90/0/Al]_T$
$A_S B_0 D_S$



GLARE 6A $[Al/45/-45/Al/-45/45/Al]_T$
$A_S B_0 D_F$

Standard FML designs with Aluminum and E-Glass/Epoxy

Governing ABD matrix of CLPT

$$\begin{Bmatrix} N_x \\ N_y \\ N_{xy} \end{Bmatrix} = \begin{bmatrix} A_{11} & A_{12} & A_{16} \\ & A_{22} & A_{26} \\ sym & & A_{66} \end{bmatrix} \begin{Bmatrix} \varepsilon_x \\ \varepsilon_y \\ \gamma_{xy} \end{Bmatrix} + \begin{bmatrix} B_{11} & B_{12} & B_{16} \\ & B_{22} & B_{26} \\ sym & & B_{66} \end{bmatrix} \begin{Bmatrix} \kappa_x \\ \kappa_y \\ \kappa_{xy} \end{Bmatrix}$$

$$\begin{Bmatrix} M_x \\ M_y \\ M_{xy} \end{Bmatrix} = \begin{bmatrix} B_{11} & B_{12} & B_{16} \\ & B_{22} & B_{26} \\ sym & & B_{66} \end{bmatrix} \begin{Bmatrix} \varepsilon_x \\ \varepsilon_y \\ \gamma_{xy} \end{Bmatrix} + \begin{bmatrix} D_{11} & D_{12} & D_{16} \\ & D_{22} & D_{26} \\ sym & & D_{66} \end{bmatrix} \begin{Bmatrix} \kappa_x \\ \kappa_y \\ \kappa_{xy} \end{Bmatrix}$$

For **E**xtensionally **I**sotropic **L**aminates:
$A_{11} = A_{22}$ and $A_{66} = (A_{11} - A_{12})/2$

For **F**ully **I**sotropic **L**aminates:
$D_{ij} = A_{ij}\, H^2/12$

For **FML**: Properties may be Extensionally Isotropic, but:
$D_{ij} \propto A_{ij}\, H^2/12$

Modulus invariants

$$U_1 = (3Q_{11} + 3Q_{22} + 2Q_{12} + 4Q_{66})/8$$
$$U_2 = (Q_{11} - Q_{22})/2$$
$$U_3 = (Q_{11} + Q_{22} - 2Q_{12} - 4Q_{66})/8$$
$$U_4 = (Q_{11} + Q_{22} + 6Q_{12} - 4Q_{66})/8$$
$$U_5 = (Q_{11} + Q_{22} - 2Q_{12} + 4Q_{66})/8$$

Q_{ij} - the reduced stiffness matrix elements

For **E**quivalent **F**ully **I**sotropic Laminate:

$$E_{Iso} = 2(1 + \nu_{Iso})G_{Iso} = U_1(1 - \nu_{Iso}^2)$$
$$\nu_{Iso} = U_4/U_1$$
$$G_{Iso} = U_5$$

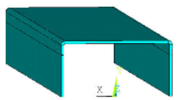
$$A_{Iso} = A_{11} = A_{22} = E_{Iso}H/(1 - \nu_{Iso}^2) = U_1H$$
$$A_{12} = \nu_{Iso}A_{11} \qquad A_{66} = U_5H$$

$$D_{Iso} = E_{Iso}H^3/(1 - \nu_{Iso}^2)/12 = U_1H^3/12$$

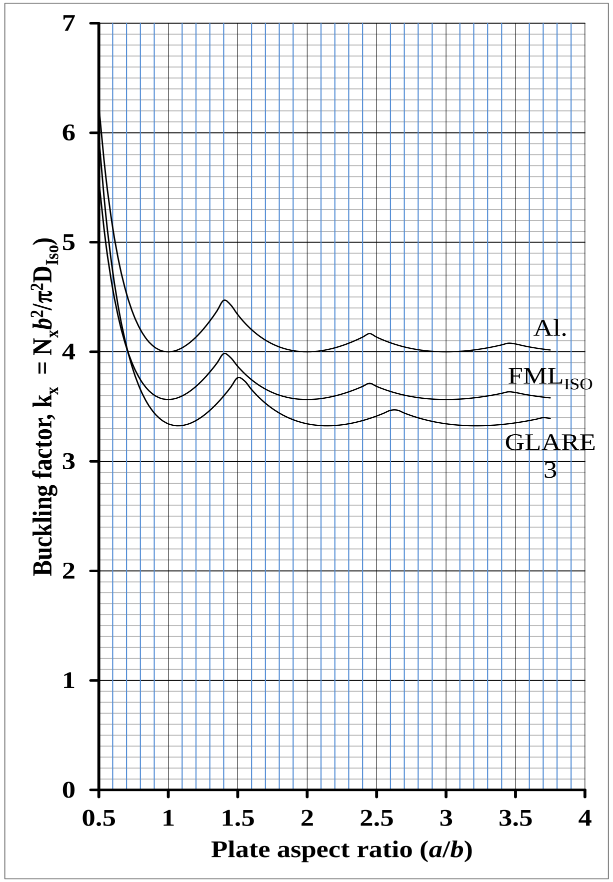
FML 8 - D_{Iso} for Aluminum = 49,391 N.mm

FML 9 - D_{Iso} for FML = 44,014 N.mm, but $D_{ij} \neq A_{ij} H^2/12$

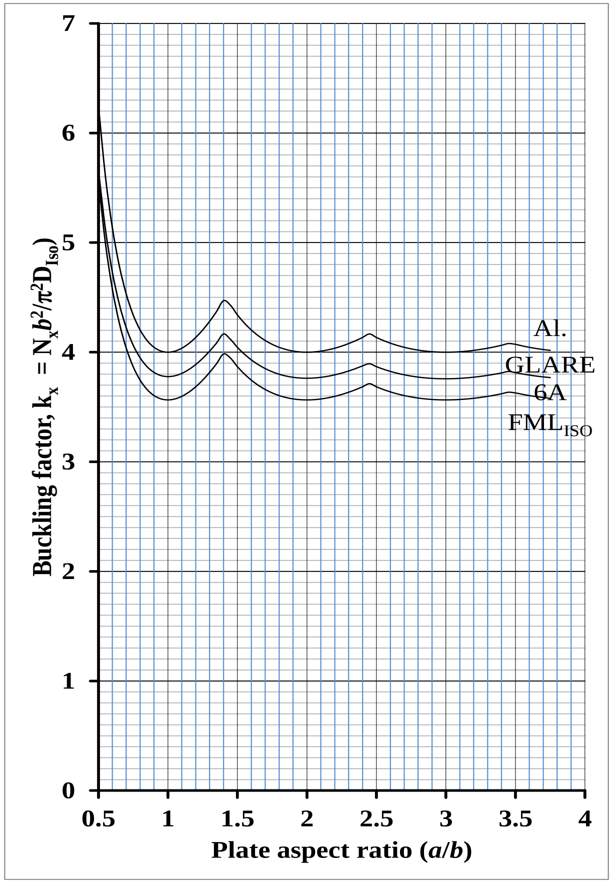
Buckling force - GFRP versus CFRP

Buckling force – channel section				
 Lay-up	GFRP	Alu	CFRP	Alu
	[kN]	reduction	[kN]	reduction
AL/0/90/AL/90/0/AL	**30.189**	0.746	**31.722**	0.783
AL/90/0/AL/0/90/AL	**29.871**	0.738	**31.132**	0.769
AL/45/0/AL/0/45/AL	**31.399**	0.776	**35.164**	0.868
AL/0/45/AL/45/0/AL	**30.588**	0.756	**33.015**	0.816
AL/0/0/AL/0/0/AL	**30.310**	0.749	**31.979**	0.790
AL/25/0/AL/0/25/AL	**30.745**	0.760	**34.241**	0.846
AL/0/25/AL/25/0/AL	**30.977**	0.765	**32.540**	0.804
Al/Al/Al/Al/Al/Al/Al	**40.472**	1.000	**40.472**	1.000
Al/*Iso*/*Iso*/Al/*Iso*/*Iso*/Al	**30.805**	0.761	**35.928**	0.888
Al/45/-45/Al/-45/45/Al	**31.752**	0.785	**36.279**	0.896

Buckling factor curves for rectangular plate



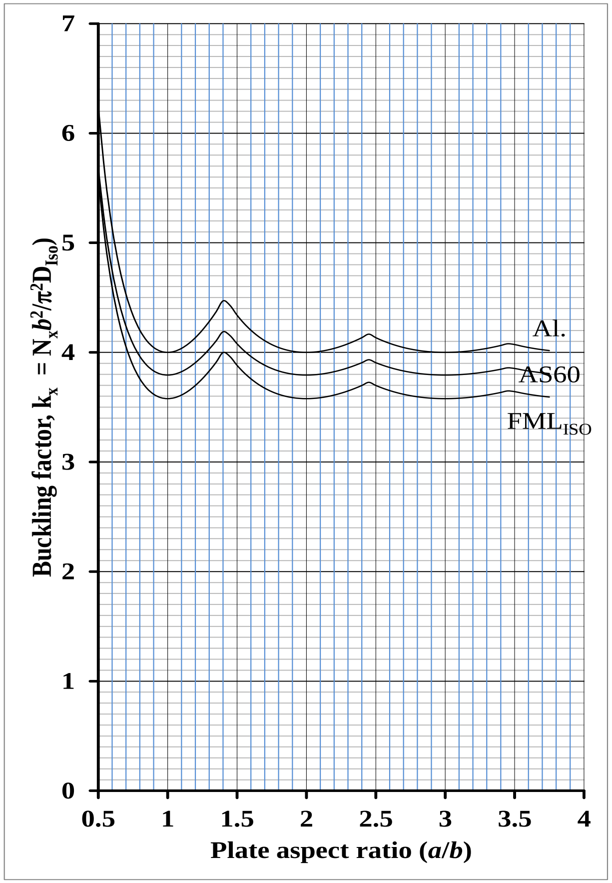
‘GLARE 3’ $[Al/0/90/Al/90/0/Al]_T$

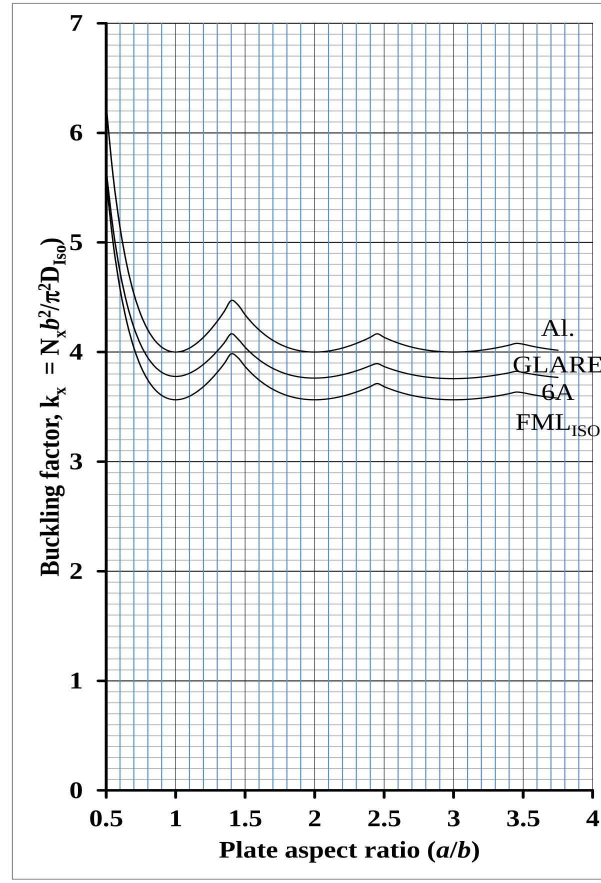


‘GLARE 6A’ $[Al/45/-45/Al/-45/45/Al]_T$.

Standard FML designs with Aluminum and Carbon/Epoxy

Buckling factor curves – NORTH ply FML





GLARE 6A $[Al/45_{12}/-45_{12}/Al/-45_{12}/45_{12}/Al]_T$

$[Al/\pm45_2/-45_2/45_2/\pm45_2/Al/\pm45_2/-45_2/45_2/\pm45_2/Al]_T$

NORTH ply FML designs with Aluminum and Carbon/Epoxy

Lamination parameters

$A_{11} = \{U_1 + \xi_1 U_2 + \xi_2 U_3\} \times H$
$A_{12} = A_{21} = \{-\xi_2 U_3 + U_4\} \times H$
$A_{22} = \{U_1 - \xi_1 U_2 + \xi_2 U_3\} \times H$
$A_{66} = \{-\xi_2 U_3 + U_5\} \times H$

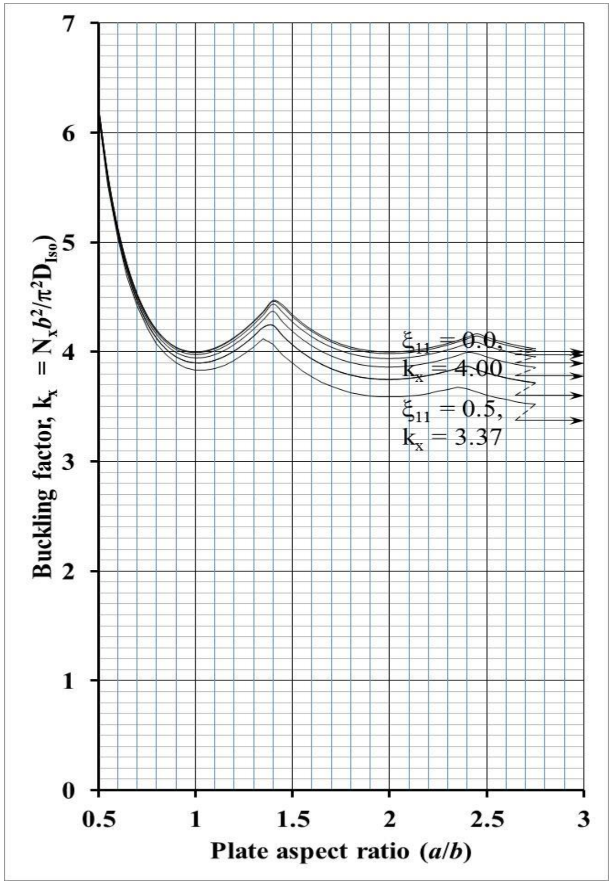
$D_{11} = A_{11} \times H^2/12$
$D_{12} = D_{21} = A_{12} \times H^2/12$
$D_{16} = D_{61} = \{\xi_{11} U_2/2\} \times H^3/12$
$D_{22} = A_{22} \times H^2/12$
$D_{26} = D_{62} = \{\xi_{11} U_2/2 - \xi_{12} U_3\} \times H^3/12$
$D_{66} = A_{66} \times H^2/12$

$$\xi^A_{\{1,2,3,4\}} = \frac{1}{h}\int_{-h/2}^{h/2} \{\cos(2\theta(z)), \cos(4\theta(z)), \sin(2\theta(z)), \sin(4\theta(z))\} dz$$

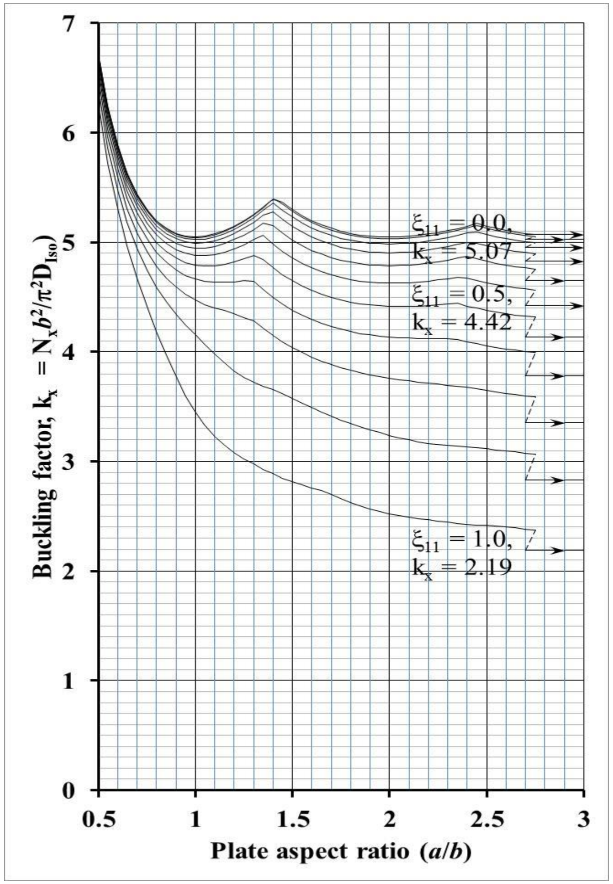
$$\xi^B_{\{1,2,3,4\}} = \frac{1}{h}\int_{-h/2}^{h/2} \{\cos(2\theta(z)), \cos(4\theta(z)), \sin(2\theta(z)), \sin(4\theta(z))\} z dz$$

$$\xi^D_{\{1,2,3,4\}} = \frac{1}{h}\int_{-h/2}^{h/2} \{\cos(2\theta(z)), \cos(4\theta(z)), \sin(2\theta(z)), \sin(4\theta(z))\} z^2 dz$$

Buckling factor curves for rectangular plate
(quasi-homogenous)



16%

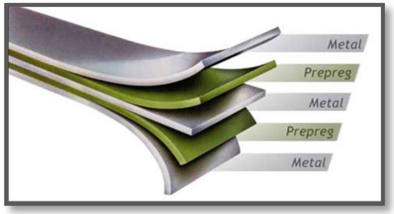


57%

quasi-isotropic laminates with (ξ_9, ξ_{10}) = (0,0) and $0 \le \xi_{11} \le 0.5$

angle-ply laminates with (ξ_9, ξ_{10}) = (0,-1) and $0.0 \le \xi_{11} \le 1.0$

Thin ply sandwich FML



Laminates possessing **F**ully **I**sotropic properties are very few in number:

36 with (n =) 18 plies ($\pi/3$ isotropy)
1 with (n =) 24 plies ($\pi/4$ isotropy)

For GLARE 2, 3 and 6:
t_{FML} = 1.9 mm; t_{Al} = 0.3 mm; t_C = 0.5 ÷ 0.25 mm (n = 2; 300gsm)

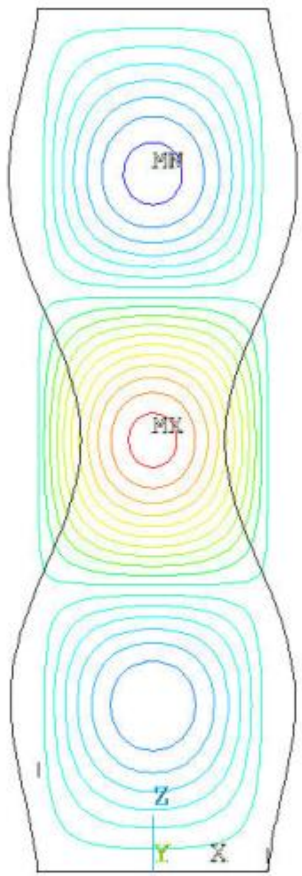
For thin ply sandwich FML: t_C = 0.5 ÷ 0.01 mm ($n \approx 24$; 30gsm)

$\mathbf{A}_I\mathbf{B}_0\mathbf{D}_I$ with n = 24:
$[-45/90/0/45/0/45/90/45/-45/0/-45/90/-45/90/45/90/0/-45/0/45/0/45/-45/90]_T$

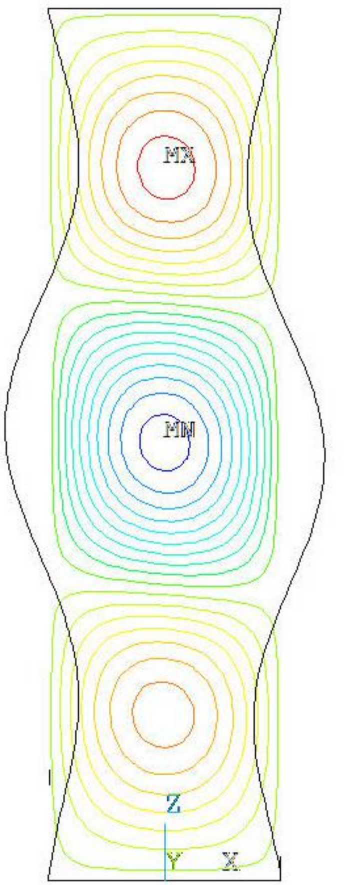
For buckling comparison the following 12 ply Quasi-Homogenous Orthotropic sub-laminate is used $[\pm 45_2/-45_2/45_2/\pm 45_2]_T$
(with 60gsm material): $[Al/\pm 45_2/-45_2/45_2/\pm 45_2/Al/\pm 45_2/-45_2/45_2/\pm 45_2/Al]_T$
and GLARE 6A $\rightarrow$ $[Al/45_{12}/-45_{12}/Al/-45_{12}/45_{12}/Al]_T$.

Engineering Sciences Data Unit, “Stiffnesses of laminated plates”, ESDU No. 94003, 1994

Buckling mode – web deflection



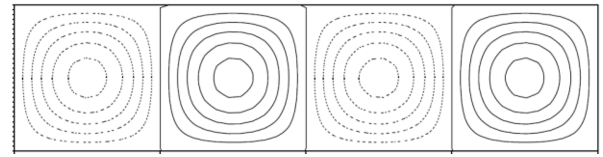
AL/0/90/AL/90/0/AL



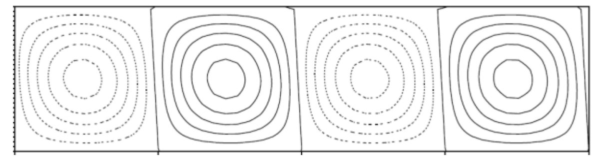
Al/45/-45/Al/-45/45/Al

Buckling mode – long plate $f(\xi_{10}, \xi_{11})$

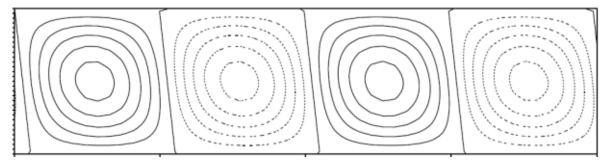
(a) $\xi_{11} = 0.0$, $k_{x,\infty} = 4.00$ and $\lambda = b$



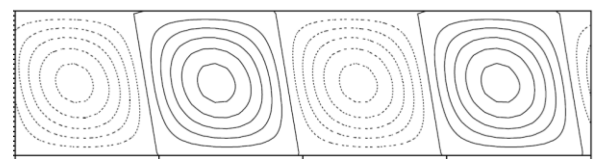
(b) $\xi_{11} = 0.1$, $k_{x,\infty} = 3.98$
and $\lambda = b$



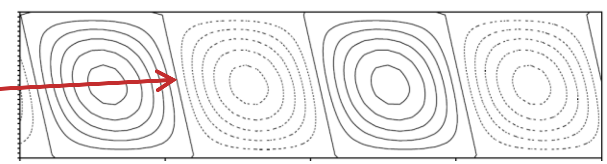
(c) $\xi_{11} = 0.2$, $k_{x,\infty} = 3.98$
and $\lambda = (298/300)b$



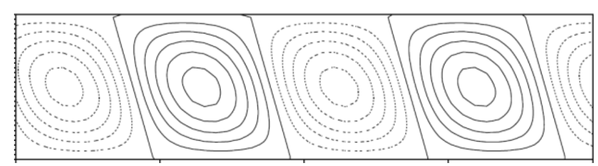
(d) $\xi_{11} = 0.3$, $k_{x,\infty} = 3.78$
and $\lambda = (296/300)b$



(e) $\xi_{11} = 0.4$, $k_{x,\infty} = 3.61$
and $\lambda = (292/300)b$



(f) $\xi_{11} = 0.5$, $k_{x,\infty} = 3.37$
and $\lambda = (286/300)b$



References

1. ESDU. Stiffnesses of laminated plates. Engineering sciences data unit, Item no. 94003; 1994.

2. Mania J.R., Kolakowski Z., Bienias J., Jakubczak P., Majerski K., Comparative study of FML profiles buckling and postbuckling behaviour under axial loading, Composite Structures, Vol. 134, 2015, pp. 216-225.

3. Tsai, W. T., Hahn, H. T., Introduction to Composite Materials, Technomic Publishing Co. Inc., Lancaster, 1980.

4. York C.B., Influence of bending–twisting coupling on compression and shear buckling strength, Stability of Structures 14th Symposium, Zakopane, Poland, 8-12 Jun 2015.

Acknowledgments

This study is supported by the Ministry of Science and Higher Education in Poland – National Science Centre Grant No **UMO-2012/07/B/ST8/04093**.

Conclusions

- **Employed three analysis methods (exp, ANM, FEM) of buckling and post-buckling response of FML profiles gave results of acceptable agreement**

- **The buckling response of considered thin-walled FML panels is dominated by metallic aluminium component** (≈46% v.f.)

- **Application of CFRP leads to lower critical force reduction with respect to aluminium but in a wider value range for considered stacking sequences than for GFRP**

Acknowledgments
This study is supported by the Ministry of Science and Higher Education in Poland – National Science Centre Grant No **UMO-2012/07/B/ST8/04093**.

Conclusions

- Thin plies allow more flexible lay-up 'tailoring' and greater homogeneity of a hybrid laminate

- Thin ply sub-laminates can also include C-Ply and TeXtreme architectures and provide a range of different mechanical properties, all within the design thickness constraints of standard FML

- Volume fractions of the two phases have a significant effect on the FML properties and need further investigation in the light of these new design configurations.
  Shear buckling may reveal additional benefits

Acknowledgments
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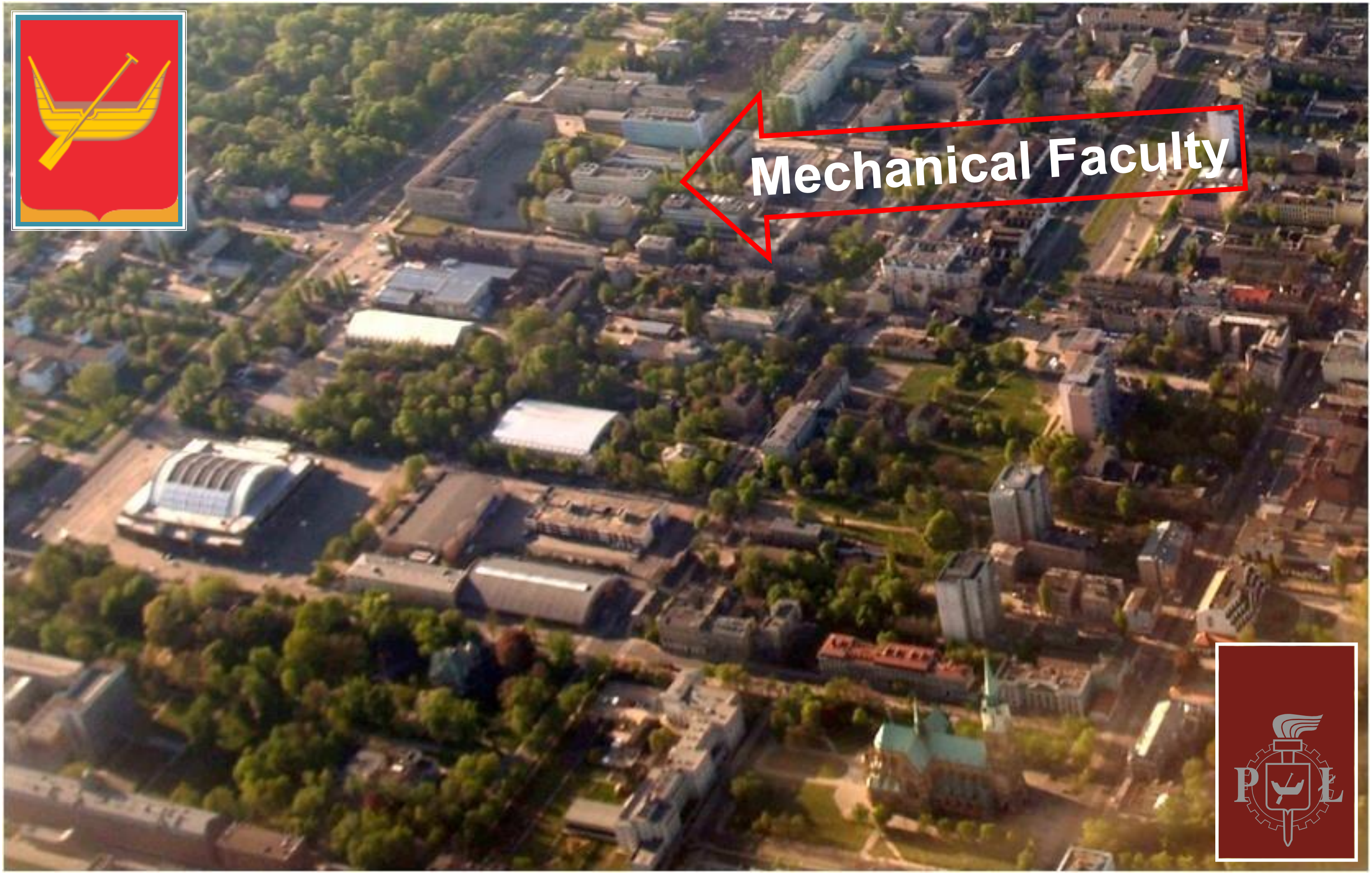

Thank you for attention